Math 202
Dr. H. Yamani

Final Exam
Summer 2016

Problem 1 (answer on pages 1,2 of the booklet) (10 pts)
Find the general solution of the ODE:

$$x^2 y'' + xy' + y = sec(\ln x)$$

Problem 2 (answer on pages 3,4 the booklet) (10 pts)
Find the general solution of the ODE:

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y} + 1$$

Problem 3 (answer on page 5 of the booklet) (10 pts)
Use Laplace transforms to solve the following:

$$y'' + y = u(t-1) + \delta(t-1) \qquad y(0) = 0 \quad y'(0) = 0$$

Problem 4 (answer on page 6 of the booklet (10 pts)
Solve the following integral equation

$$y'(t) = 1 - t - \int_0^t y(\tau)d\tau \qquad y(0) = 0$$

Problem 5 (answer on pages 7, 8 of the booklet) (10 pts each)
Find the general solution of the following systems.

a. $X' = \begin{bmatrix} 3 & -1 \\ 9 & -3 \end{bmatrix} X$ **b.** $X' = \begin{bmatrix} 4 & -5 \\ 5 & -4 \end{bmatrix} X$

Problem 6 (answer on page 9 of the booklet) (10 pts)
Use Green's theorem to find the counterclockwise circulation and outward flux of the field $\mathbf{F} = \left(x + e^x \sin y\right)\mathbf{i} + \left(x + e^x \cos y\right)\mathbf{j}$ around and across $r = 2(1 + \cos\theta)$.

Problem 7 (answer on pages 10 ,11 of the booklet) (10 pts)
Show that the differential form in the following integral is exact, then evaluate the integral

$$\int_{(3,-2,0)}^{(1,0,\pi)} \left(2x\cos z - x^2\right) dx + (z - 2y)\, dy + \left(y - x^2 \sin z\right) dz$$

Problem 8 (answer on pages 12, 13 of the booklet) (10 pts)
Use Stokes' theorem to evaluate the counterclockwise circulation of the field $\mathbf{F} = y^2\boldsymbol{i} - y\boldsymbol{j} + 3z^2\boldsymbol{k}$ around the boundary C which is the intersection of the plane $2x + 6y - 3z = 6$ and the cylinder $x^2 + y^2 = 1$

Problem 9 (answer on pages 14, 15 of the booklet) (10 pts)
Use the divergence theorem to find the outward flux of the field $\mathbf{F} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ across the boundary of the region which is the surface of the upper cap cut from the solid sphere $x^2 + y^2 + z^2 \le 25$ by the plane $z = 3$